INTERNATIONAL SCIENTIFIC JOURNAL OF BIRUNI
ISSN (E) 2181-2993
Vol. 2, Issue 2. May. (2023)

STUDY OF THE LAWS OF NATURAL VIBRATIONS OF VISCOELASTIC CYLINDRICAL SHELLS REINFORCED WITH VISCOELASTIC FILLER

AKHMEDOV Sh.R.
Candidate of technical sciences, Professor of Bukhara Institute of Natural Resources Management, National Research University "Tashkent Institute of Irrigation and Agricultural Mechanization Engineers".
The Republic of Uzbekistan, 105017, Gazli street 32,
ahmedovsarifboj652@gmail.com

TOYIROV M.Z,
Bachelor student of Bukhara Institute of Natural Resources Management, National Research University "Tashkent Institute of Irrigation and Agricultural Mechanization Engineers".
The Republic of Uzbekistan, 105017, Gazli street 32,
toirovmuhriddin82@gmail.com

https://doi.org/10.24412/2181-2993-2023-2-236-244

ABSTRACT

Reinforced viscoelastic cylindrical shells with deformable fillers are used in rocketry, aircraft construction, shipbuilding, and construction. To give greater rigidity, the thin-walled part of the shell is reinforced with ribs, while a slight increase in the weight of the structure significantly increases its strength, even if the ribs are of low height. The purpose of this work is to develop a technique, algorithm and programs for finding resonant frequencies and modes of vibration for circular ribbed viscoelastic cylindrical shells under various boundary conditions. The paper deals with vibrations of longitudinally reinforced cylindrical shell structures with a filler. The variation principle is used to study the vibrations of a thin longitudinally reinforced viscoelastic cylindrical shell under dynamic influences. Oscillatory processes of the filler and the bonded shell satisfy the Lame equations. At the contact between the shell and the filler, the conditions of rigid contact are fulfilled. The relationship between stresses and strains, for a linear viscoelastic material, is represented in the form of the Boltzmann-Voltaire integral.

Key words: shell, instantaneous modulus of elasticity, deformation, total energy, ribs, viscosity.

1.Introduction.

Reinforced viscoelastic cylindrical shells with deformable fillers are widely used in rocketry, aircraft construction, shipbuilding, and construction. To give greater rigidity, the thin-walled part of the shell is reinforced with ribs, while a slight increase in the weight of the structure significantly increases its strength, even if the ribs are of low height. The study and elimination of resonance phenomena in shells is of great practical interest. A large number of theoretical and experimental works are devoted to the study of natural vibrations of circular cones. However, there are still no reliable solutions that allow determining the parameters of resonances in a wide range of changes in physical and geometric parameters. There are also works in which the theoretical and experimental method obtained dependences for determining the resonance frequencies [1] and vibration modes of truncated conical panels [2,3]. Another method is mainly used to study shells, which allow one to go from the stability equations for conical shells to the corresponding equations for cylindrical shells with a circular cross section. Many works use moment-free and semi-momentless shell theories [4, 5]. Approximate methods are also used to solve problems of natural vibrations [6,7].

2. Methods

2.1. Problem statement and basic relations

This work is devoted to the study of free vibrations of cylindrical shells with a filler, reinforced by discretely distributed longitudinal systems of ribs under axial compression and taking into account the friction between the shell and the filler. The analysis of the influence of the parameters of the external environment on the parameters of the frequency of natural vibrations of the system is carried out. The problem was solved in an energetic way. The potential energy of a shell loaded with axial compressive forces has the form [8]:

Consider a closed circular cylindrical shell with *R* radius, thickness, with edges *k* and *n* (along the longitudinal and annular directions, respectively). To obtain the equations of natural vibrations, we use the Lagrange principle of possible displacements, which takes into account the boundary conditions

$$\delta(K+\Pi-A)=0 \quad , \qquad (1)$$

where K is the kinetic energy of the shell and the rib, P is the potential energy of the shell and the rib, A is the work of external forces.

The problem was solved in an energetic way. The potential energy of a shell loaded with axial compressive forces has the form [8]:

$$\Pi=\frac{Eh}{2(1-\nu^2)}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial u}{\partial\xi}+\frac{\partial v}{\partial\theta}-w\right)^2+2(1-\nu)\frac{\partial u}{\partial\xi}\left(\frac{\partial v}{\partial\theta}-w\right)-\right.$$

$$\left.-\frac{1}{4}\left(\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\Bigg\}d\xi d\theta+\frac{Eh}{24(1-\nu^2)R^2}\int_0^{\xi_1}\int_0^{2\pi}\left(\frac{\partial^2 w}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)^2-$$

$$-2(1-\nu)\left[\frac{\partial^2 w}{\partial\xi^2}\left(\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)-\frac{1}{4}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\Bigg\}d\xi d\theta+$$

$$+\frac{E_c}{2R}\sum_{l=1}^{k}\int_0^{\xi_1}\left[F_c\left(\frac{\partial u}{\partial\xi}-\frac{h_c}{R}\frac{\partial^2 w}{\partial\xi^2}\right)^2+\frac{I_{yc}}{R^2}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2+\frac{G_c}{E_c}I_{kp.c}\times\frac{G_c}{E_c}I_{kp.c}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]_{\theta=\theta_1}d\xi-$$

$$-\frac{\sigma_x h}{2}\int_0^{\xi_1}\int_0^{2\pi}\left(\frac{\partial w}{\partial\xi}\right)^2 d\xi d\theta-\frac{\sigma_x F_c}{2R}\sum_{i=1}^{k}\int_0^{\xi_1}\left(\frac{\partial w}{\partial\xi}\right)^2\Bigg|_{\theta=\theta_1}d\xi$$

here $\xi_1=\frac{L}{R},\ \xi=\frac{x}{R},\ \theta=\frac{y}{R}$; x, y, z- coordinates, E_c, G_c- elastic moduli and shears of the material of the longitudinal ribs, k – number of longitudinal ribs, σ_x - axial compressive stresses, u, v, w - components of the shell displacement vector, E, ν - Young's modulus and Poisson's ratio of the shell material, F_c, I_{yc}, $I_{kp.c}$ - respectively, the areas and moments of inertia of the cross-section of the longitudinal bar relative to the axes *OX* и *OZ,* and also the moment of inertia during torsion.

The interaction of the filler with the shell is represented as a surface load applied to the shell, which performs work on the displacements of the contact surface during the transfer of the system from the deformed state to the initial undeformed state

$$A_0 = -\int_0^{\xi_1}\int_0^{2\pi}(q_x u + q_\theta v + q_z w)d\xi d\theta + \int_0^{\xi_1}\int_0^{2\pi} f q_z (u+v)d\xi d\theta \qquad (2)$$

where q_x, q_θ, q_z - pressure from the filler on the shell, f is the friction coefficient. Total system energy: $П = Э + К + A_0$.

Physical relations for an isotropic viscoelastic body take the form [9]

$$\sigma_x = \frac{\tilde{E}}{1-\nu^2}\left(\varepsilon_x^z + \nu\varepsilon_y^z\right);\ \sigma_y = \frac{\tilde{E}}{1-\nu^2}\cdot\left(\varepsilon_y^z + \nu\varepsilon_x^z\right);$$

$$\tau_{xy} = \frac{\tilde{E}}{2(1+\nu)}\gamma_{xy}^z;\tau_{xz} = \frac{\tilde{E}}{2(1+\nu)}\gamma_{xz}\ ;\ \tau_{yz} = \frac{\tilde{E}}{2(1+\nu)}\gamma_{yz}.$$

Here μ - Poisson's ratio of the shell material, which is assumed to be constant; $\tilde{E}_к$ - operator moduli of elasticity of the conical shell and ribs

$$\tilde{E}_k\left[f(t)\right] = E_{0\kappa}\left[f(t) - \int_0^t R_{E\kappa}(t-\tau)f(\tau)d\tau\right]\ , \qquad (3)$$

$E_{0\kappa}$ - Young's instant modulus (κ=1,2,3...L); $\kappa=1$- instantaneous modulus of elasticity of the shell, $\kappa=2,3...L$– instant modulus of elasticity of ribs, $f(t)$ - continuous function; $R_{E\kappa}(t-\tau)$ - relaxation core.

Physical relations, taking into account the creep of the material (3) on the basis of the linear theory of heredity, take the form [10]

$$\sigma_x = \frac{E_0}{1-\nu^2}\left[\varepsilon_x^z + \nu_1\varepsilon_y^z - \int_0^t(\varepsilon_x^z + \nu_1\varepsilon_y^z)R_{E1}(t-\tau)d\tau\right];$$

$$\sigma_y = \frac{E_0}{1-\nu^2}\left[\varepsilon_y^z + \nu_1\varepsilon_x^z - \int_0^t(\varepsilon_y^z + \nu_1\varepsilon_x^z)R_{E1}(t-\tau)d\tau\right];$$

$$\tau_{xy} = \frac{E_0}{2(1+\nu)}\left[\gamma_{xy}^z - \int_0^t \gamma_{xy}^z R_{E2}(t-\tau)d\tau\right];$$

$$\tau_{zx} = \frac{E_0}{2(1+\nu)}\left[\gamma_{zx}^z - \int_0^t \gamma_{zx}^z R_{E2}(t-\tau)d\tau\right], \qquad (4)$$

$$\tau_{yz} = \frac{E_0}{2(1+\nu)}\left[\gamma_{yz}^z - \int_0^t \gamma_{yz}^z R_{E2}(t-\tau)d\tau\right].$$

Here $R_{E1}(t-\tau), R_{E2}(t-\tau)$- relaxation nuclei. The influence of the stiffness of the ribs is taken into account using the Dirac impulse function.

The location and height of the ribs is set by the function

$$H(x,y)=\sum_{j=1}^{m}h^{j}\,\bar{\delta}(x-x_{j})+\sum_{i=1}^{n}h^{i}\,\bar{\delta}(y-y_{i})-\sum_{i=1}^{n}\ \sum_{j=1}^{m}h^{ij}\,\bar{\delta}(x-x_{j})\bar{\delta}(y-y_{i}) \quad (5)$$

Integrating voltages (3) over z, ranging from $-\frac{h}{2}$ до $\frac{h}{2}+H$, we obtain the forces, moments and transverse forces reduced to the middle surface of the shell, for a unit length of the middle surface

$$N_x=\tilde{G}_1\left[\left(h+\bar{F}\right)\cdot\varepsilon_1+\bar{S}\psi_1\right];$$

$$N_y=\tilde{G}_2\left[\left(h+\bar{F}\right)\cdot\varepsilon_2+\bar{S}\psi_2\right];$$

$$N_{xy}=\tilde{G}_{12}\left[\left(h+\bar{F}\right)\gamma_{xy}+\bar{S}\psi_{12}\right];$$

$$M_x=\tilde{G}_1\left[\bar{S}\,\varepsilon_1+\left(\frac{h^3}{12}+\bar{J}\right)\psi_1\right];$$

$$M_y=\tilde{G}_2\left[\bar{S}\,\varepsilon_2+\left(\frac{h^3}{12}+\bar{J}\right)\cdot\psi_2\right], \quad (6)$$

$$M_{xy}=\tilde{G}_{12}\left[\bar{S}\,\gamma_{xy}+\left(\frac{h^3}{12}+\bar{J}\right)\psi_{12}\right];$$

$$Q_x=k\tilde{G}_{13}\left(h+\bar{F}\right)\cdot\left(\psi_x+\frac{\partial W}{\partial x}\right),$$

$$Q_y=k\tilde{G}_{23}\left(h+\bar{F}\right)\left(\psi_y+\frac{1}{x\sin\theta}\frac{\partial W}{\partial y}+\frac{\operatorname{ctg}\theta}{x}V\right),$$

Where

$$\varepsilon_1=\varepsilon_{xx}+\nu\varepsilon_{yy},\ \varepsilon_2=\varepsilon_{yy}+\nu\varepsilon_{xx},\ \psi_1=\chi_1+\nu\chi_2,\ \psi_2=\chi_2+\nu\chi_1,\ \psi_{12}=2\chi_{12},$$

$$\tilde{G}_1[f(t)]=\tilde{G}_2[f(t)]=\frac{\tilde{E}}{1-\nu^2}[f(t)]=$$

$$=\frac{E_0}{1-\nu^2}\left(f(t)-\int_0^t R_E(t-\tau)f(\tau)d\tau\right),$$

$$\tilde{G}_{12}[f(t)]=\tilde{G}_{13}[f(t)]=\tilde{G}_{23}[f(t)]=\frac{\tilde{E}[f(t)]}{2(1+\nu)}=;$$

$$=\frac{E_0}{2(1+\nu)}\left(f(t)-\int_0^t R_E(t-\tau)f(\tau)d\tau\right)$$

$\bar{F}$, $\bar{S}$, $\bar{J}$ — areas (cross-section or longitudinal) of the ribs per unit length of the median surface. The static moment and the moment of inertia of the middle surface of the shell have the form

$$\overline{F} = \int_{h/2}^{h/2+H} dz \quad ; \quad \overline{S} = \int_{h/2}^{h/2+H} z\,dz; \; \overline{J} = \int_{h/2}^{h/2+H} z^2 dz.$$

Let the transverse dynamic load act on the shell $q(x,y,t)$. Then the unknown unknown displacement functions U,V,W and the angles of rotation of the normal ψ_x, ψ_y are functions of variables x, y и t. The functional of the total strain energy of the viscoelastic shell has the form

$$J = \int_{t_0}^{t_1} (K - \Pi + A)dt \quad . \qquad (7)$$

Shell kinetic energy:

$$K = \frac{Eh}{2(1-\nu^2)} \int_0^{\xi_1}\int_0^{2\pi} \left[\left(\frac{\partial u}{\partial t_1}\right)^2 + \left(\frac{\partial \nu}{\partial t_1}\right)^2 + \left(\frac{\partial w}{\partial t_1}\right) \right] d\xi d\theta$$

$$+ \frac{\overline{\rho_c} E_c F_c}{2R(1-\nu^2)} \sum_{i=1}^{k_1} \int_0^{\xi_1} \left[\left(\frac{\partial u}{\partial t_1}\right)^2 + \left(\frac{\partial w}{\partial t_1}\right) \right]_{\theta=\theta_1} d\xi \qquad (8)$$

here $\overline{\rho_c} = \frac{\rho_c}{\rho_0}$, where ρ_0, ρ_c - the density of the shell and rod materials, respectively, $\theta_i = \frac{2\pi}{k_1} i$.

The equation of motion of the medium, in vector form, has the form [11,12]:

$$a_e^2 qrad\, div \vec{S} - a_t^2 rot\, rot \vec{S} + \omega^2 \vec{S} = 0 \quad , 0 \le x \le L,\ 0 \le r \le R \qquad (9)$$

where $a^2{}_t = (\lambda + 2\mu)/\rho$, $a^2{}_e = \mu/\rho$, a_t, a_e - velocities of propagation of longitudinal and transverse waves in the aggregate, respectively; $S = S(S_x, S_\theta, S_z)$ - displacement vector; λ, μ - Lame coefficients. Contact conditions are added to the systems of equations of motion of the medium (9). It is assumed that the contact between the shell and the core is rigid, i.e. at $r = R$:

$$u = S_x;\; \nu = S_\theta;\quad w = S_z \qquad (10)$$

$$q_x = -\sigma_{rx}, q_y = -\sigma_{r\theta}, \quad q_z = -\sigma_{rr}, w = S_r \qquad (11)$$

Components $\sigma_{rx}, \sigma_{r\theta}$, σ_{rr} - stress tensors are defined as follows [13]:

$$\sigma_{rx} = \tilde{\mu}_s \left(\frac{\partial S_x}{\partial r} + \frac{\partial S_r}{\partial x} \right); \quad \sigma_{r\theta} = \tilde{\mu}_s \left[r \frac{\partial}{\partial r} (\frac{S_r}{r}) + \frac{1}{r}\frac{\partial S_r}{\partial \theta} \right],$$
$$\sigma_{rr} = \tilde{\lambda}_s \left(\frac{\partial S_r}{\partial x} + r\frac{\partial}{\partial r}(\frac{S_r}{r}) + \frac{1}{r}\frac{\partial S_\theta}{\partial \theta} \right) + 2\tilde{\mu}_s \frac{\partial S_r}{r} \qquad (12)$$

where $\tilde{\lambda}_s$, $\tilde{\mu}_s$ -operator Lamé coefficients for a medium in the form (2).

Supplementing the equations of motion of the filler (9) with contact conditions (11) and (12), we arrive at the contact problem of vibrations of a cylindrical shell, supported by cross systems of ribs filled with a medium. In other words, the problem of vibrations of the ribs of a cylindrical shell with a filler reinforced by cross systems under axial compression is reduced to the joint integration of the equations of the theory of shells and the equations of motion of the filler when the specified conditions are met on the surface of their contact.

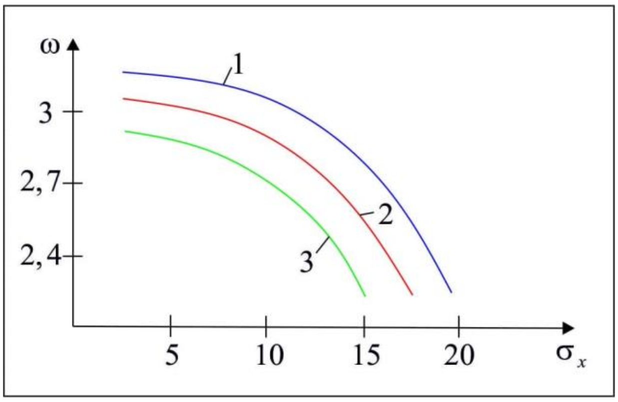

Fig. 1. System frequency dependence $\omega = \omega_1\omega_0$ from compressive stresses at different values *n*: 1. *n*=2; 2. *n*=3; 3. *n*=4.

As an example to illustrate the change μ depending on the relative weights of the ribs, the results of calculations of the ribs of cylindrical shells filled with a medium, reinforced by longitudinally reinforced systems, are presented.

The calculation results are shown in Fig. 2 as curves

$\mu(\eta_1) = \frac{\omega_R^2}{\nu^2}$ для различных значений η_2. Solid curves correspond to vibrations of a ribbed shell without fillers.

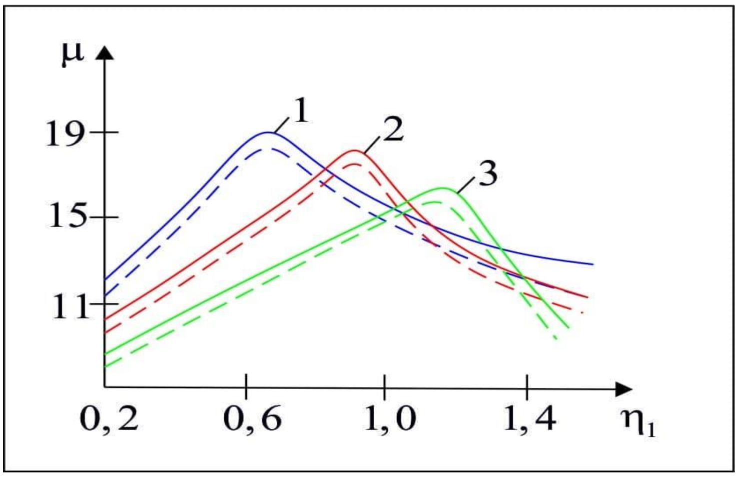

Fig. 2. Change in the dynamism coefficient depending on the relative weights of the ribs at different weights of the transverse rib1. 1. $\eta_1 = 0$; 2. $\eta_1 = 0,2$; 3. $\eta_1 = 0,4$.

Analyzing the results of calculations, it is easy to see that the best bearing capacity of the shell is achieved with reinforcement only by transverse ribs (η_2=0), for whom $\mu_{max} = 18,7031$. The abscissa of this point is 0,50.

5. Conclusions

1. An algorithm for solving the problems of natural vibrations of shells for ribbed viscoelastic cylindrical shells has been developed. To solve dynamic problems, the method of separation of variables and special functions of the subject of the equations of mathematical physics, the freezing method, the methods of Mueller and Gauss are used.

2. Analysis of the calculation results shows that with a decrease in the thickness of the viscoelastic cylindrical shell, the real and imaginary parts of the first and second vibration frequencies decrease monotonically. The real parts of the third and fourth frequencies decrease moderately, while the corresponding imaginary parts increase smoothly.

3. Taking into account the rheological properties of the material makes it possible to increase (or decrease) the frequency values of the shell up to 10%.

REFERENCES

1. Lal, R. Transverse vibrations of orthotropic non-uniform rectangular plate with continuously varying density//Indian Journal of Pure and Applied Mathematics, 34, 587–606, (2003).
2. Kaplunov J.D., Wilde M.V. Edge and interfacial vibrations in elastic shells of revolution // J. Appl. Math. Phys. (ZAMP). 2000. - 51. - P. 29-48.
3. Латифов Ф.С.. Колебания оболочек с упругой и жидкой средой. Баку, «Элм», 1999, 164с.
4. Alexey A. Semenov. Model of deformation stiffened orthotropic shells under dynamic loading. Journal of Siberian Federal University. Mathematics and Physic 2016, 9(4), p.485-497.
5. Akhmedov Sh.R., Zhuraev T.O. Modern instrumental systems, information technology and innovation. Experimental methods for studying wave propagation in half-space 2020 yil, 31-33.
6. Duskaraev N.A., Akhmedov Sh.R., Dustkaraev A.N., Zhuraev T.O. The effectiveness of the application of innovative technologies and techniques in

agriculture and water management. The problem of increasing the accuracy and productivity of machining on metal-cutting machines. 2020 yil, 485-487.

7. Akhmedov Sh.R., Zhuraev T.O. The effectiveness of the application of innovative technologies and techniques in agriculture and water management. About the of accidental non-stationary loads on the construction structure. 2020 yil, 449-451.

8. Bosyakov S.M., Chzhiwei V. Analysis of Free Vibrations of Cylindrical Shells Made of Fiberglass with Navier Boundary Conditions. Mechanics of machines, mechanisms and materials. 2011, no. 3, p. 24-27.6.F.S. Latifov, F.A. Seyfullaev, Sh. Sh. Alyev. Free vibrations reinforced by transverse ribs of an anisotropic cylindrical shell made of fiberglass with a liquid flowing in it. Applied mechanics and technical physics. 2016, Vol. 57, No. 4, pp. 158-162.

9. A.I. Seyfullayev, K. A. Novruzova, Oscillations of longitudinally reinforced orthotropic cylindrical shell filled with a viscous fluid, Eastern-European Journal of Enterprise Technologies, (2015), no. 3/7 (75), 29-33.

10. Mirsaidov M.M, Safarov I.I and Teshaev M.X, Dynamic instability of vibrations of thin-wall composite curvorine viscoelastic tubes under the influence of pulse pressure E3SWebofConferences 164 (14013) pp. 1-12. (2020)

11. Khudainazarov Sh, Mavlanov T, Qosimov J, Nurova O, Forced vibrations of high-rise buildings IOP Conf. Series: Materials Science and Engineering 869 pp. 1-